SE Sem III Etrx & Extc CBGS App. Maths - III 2/12/16.

Q. P. Code : 547400

DURATION: 3 HRS. **MAX. MARKS:80**

1) Question No. 1 is compulsory.
2) Attempt any **THREE** of the remaining.
3) **Figures** to the **right** indicate **full** marks.

Q 1.A) Determine the constants a, b, c, d, e if

$f(z) = (ax^4 + bx^2 y^2 + cy^4 + dx^2 - 2y^2) + i(4x^3 y - exy^3 + 4xy)$ is analytic. (5)

B) Find half range Fourier sine series for $f(x) = x^2$, $0 < x < 3$. (5)

C) Find the directional derivative of $\varphi(x, y, z) = xy^2 + yz^3$ at the point (2,-1,1) in the direction of the vector $i + 2j + 2k$. (5)

D) Evaluate $\int_0^\infty e^{-2t} t^5 \cosh t \; dt$. (5)

Q.2) A) Prove that $J_{\frac{3}{2}}(x) = \sqrt{\frac{2}{\pi x}} \left(\frac{\sin x}{x} - \cos x\right)$ (6)

B) If $f(z) = u + iv$ is analytic and $u - v = e^x(\cos y - \sin y)$, find f(z) in terms of z. (6)

C) Obtain Fourier series for $f(x) = x + \frac{\pi}{2}$ $\quad -\pi < x < 0$

$= \frac{\pi}{2} - x \quad 0 < x < \pi$

Hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$ (8)

Q.3) A) Show that $\bar{F} = (2xy + z^3)i + x^2 j + 3xz^2 k$, is a conservative field. Find its scalar potential and also find the work done by the force $\bar{F}$ in moving a particle from (1,-2,1) to (3,1,4). (6)

B) Show that the set of functions $\{\sin(2n+1)x\}, n = 0,1,2, \dots$ is orthogonal over $[0, \pi/2]$. Hence construct orthonormal set of functions. (6)

[TURN OVER]

C) Find (i) $L^{-1}\{\cot^{-1}(s+1)\}$

(ii) $L^{-1}\left(\frac{e^{-2s}}{s^2+8s+25}\right)$ (8)

Q.4) A) Prove that $\int J_3(x)\,dx = -\frac{2J_1(x)}{x} - J_2(x)$ (6)

B) Find inverse Laplace of $\frac{s}{(s^2+a^2)(s^2+b^2)}$ ($a \neq b$) using Convolution theorem. (6)

C) Expand $f(x) = x\sin x$ in the interval $0 \leq x \leq 2\pi$ as a Fourier series.

Hence, deduce that $\sum_{n=2}^{\infty} \frac{1}{n^2-1} = \frac{3}{4}$ (8)

Q.5) A) Using Gauss Divergence theorem evaluate $\iint_S \bar{N}.\bar{F}\,ds$ where $\bar{F} = x^2 i + zj + yzk$

and S is the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$ (6)

B) Prove that $J_2'(x) = \left(1 - \frac{4}{x^2}\right)J_1(x) + \frac{2}{x}J_0(x)$ (6)

C) Solve $(D^2+3D+2)y = 2(t^2+t+1)$, with $y(0)= 2$ and $y'(0)= 0$ by using Laplace transform (8)

Q.6) A) Evaluate by Green's theorem for $\int_c (e^{-x}\sin y\,dx + e^{-x}\cos y\,dy)$ where C is the the rectangle whose vertices are $(0,0)$, $(\pi, 0)$, $(\pi, \pi/2)$ and $(0, \pi/2)$ (6)

B) Show that under the transformation $w = \frac{z-i}{z+i}$, real axis in the z-plane is mapped onto the circle $|w| = 1$ (6)

C) Find Fourier Sine integral representation for $f(x) = \frac{e^{-ax}}{x}$ (8)

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